Numerical Modeling of Electron Beams Accelerated by the Radio Frequency Boundary Sheath

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***Abstract*—The stochastic heating of electrons by the radio frequency boundary sheath in capacitively coupled plasmas is not completely understood or at least agreed upon by researchers. To aid in understanding this phenomena, a conceptually simple simulation of electron heating is presented. A fluid model is used to calculate the electric fields in the discharge, and a Monte Carlo simulation is used to calculate electron distribution functions. The plots of the density of energetic electrons are presented in this paper. They show electron beams that have been accelerated by the sheath.**

***Index Terms*—Electron beams, plasma devices, plasma heating, plasmas, plasma sheaths.**

TO AID in understanding the stochastic heating of electrons in low pressure RF capacitively coupled plasmas (CCPs), a 1-D conceptually simple simulation is presented. The electric fields are all calculated in advance by a fluid model, and either one or a group of test electrons is integrated in time to calculate either the electron velocity distribution function or the electron energy distribution function. For the electric fields in the sheath region, the sheath model of Brinkmann was used [1], [2]. The model also includes the ohmic electric field, or the field due to conduction current, throughout the entire discharge and elastic, as well as inelastic, Monte Carlo collisions for electrons, as one would have in a Particle in a Cell (PIC) simulation. Therefore, the model simulates both the stochastic and ohmic heating mechanisms. Ohmic electron heating is the normal collisional heating of electrons in the plasma bulk, and stochastic electron heating is due to electrons interacting with spatially and temporally inhomogeneous electric fields in the RF plasma boundary sheath. The simulation does not include electron–electron Coulomb collisions. The particle and energy balance are not used to calculate the electron temperature and density; thus, it is not a self-consistent model. This model is very useful though, because one can turn on or off physical effects, which is something that one cannot easily do with a self-consistent model.

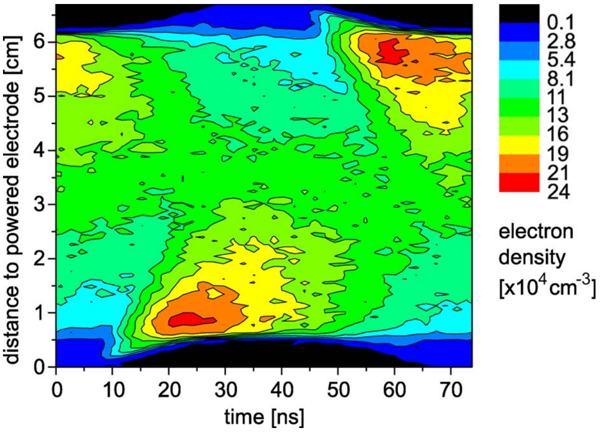

Fig. 1. Density of electrons with an energy above 12 eV in a simulated CCP discharge. Input parameters were taken from the experimental measurements of the 20-mTorr case in [3, Fig. 18]. The figure demonstrates the heating of electrons caused by an interaction with an expanding sheath.

Fig. 1 shows the density of electrons with an energy above 12 eV for a simulated symmetric CCP discharge. For input parameters, an effective electron temperature and electron density were calculated from data digitized from the 20-mTorr electron energy probability function (EEPF) in [3, Fig. 18]. The geometry of the simulation matches the experimental description contained in [3]. Comparisons between the measured EEPF and the EEPF calculated by the simulation demonstrate that the model, in fact, approximates the distribution function for a broad range of pressures and transitions from stochastic to ohmic heating at the correct pressure. This will be the subject of a future publication.

Fig. 1 shows the electrons accelerated by the plasma sheath. A larger concentration of energetic electrons is visible in front of the expanding sheath. The only way that electrons gain energy is through interaction with the ohmic electric field and the electric fields in the sheath. An additional simulation was performed to demonstrate this mechanism. Fig. 2 again shows the density of electrons with an energy above 12 eV. The identical simulation parameters were used except that an additional RF harmonic was added to modulate the sheath. In the original experiment, it is a symmetric single-frequency discharge; therefore, there are no or only very small harmonics. The sixth harmonic was set to be 10% of the fundamental's amplitude. The RMS current was adjusted to 3 $\mathrm{mA/cm^2}$ to be consistent with the original simulation shown in Fig. 1. In Fig. 2, multiple beams are now observed.

Although the discharge is symmetric, the beam structures near the grounded and powered electrodes in Fig. 2 are not

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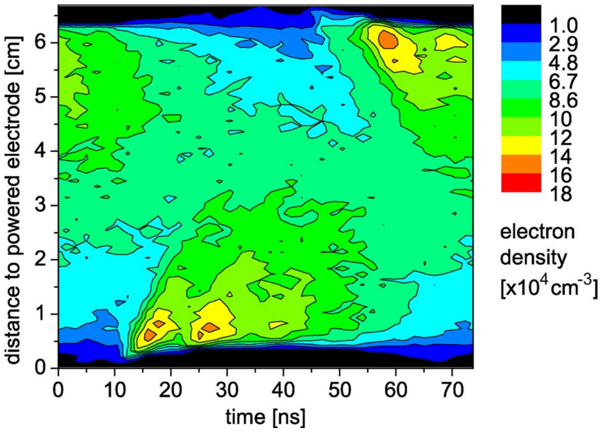

Fig. 2. Density of electrons with an energy above 12 eV for the simulation conditions shown in Fig. 1 except that a sixth harmonic has been added to the applied RF voltage. Individual electron beams are visible due to the modulation of the sheath. Asymmetry in the beam structure is due to the additional harmonic being an even harmonic.

symmetric. The total voltage applied to the discharge has the following form: $V = A_0(\cos(\omega t) + 0.1\sin(6\omega t))$ with $t = 0$ corresponding to time $= 0$ in Fig. 2. A proof that an even harmonic in the applied voltage will break the electrical symmetry of the two sheaths follows.

To begin, it is assumed that the total voltage across the discharge V_{total} is the sum of the voltage drops across the powered V_{sp} and grounded sheaths V_{sg}

$$V_{\text{total}}(\omega t) = V_{\text{sp}}(\omega t) + V_{\text{sg}}(\omega t). \tag{1}$$

This is a reasonable approximation for a symmetric discharge.

V_{total}, V_{sp}, and V_{sg} can be decomposed into its Fourier terms

$$V_{\text{total}}(\omega t) = \sum_{n=-N}^{N} t_n e^{in\omega t} \tag{2}$$

$$V_{\text{sp}}(\omega t) = \sum_{n=-\infty}^{\infty} p_n e^{in\omega t} \tag{3}$$

$$V_{\text{sg}}(\omega t) = \sum_{n=-\infty}^{\infty} g_n e^{in\omega t} \tag{4}$$

where $\omega = 2\pi f$; t_n, p_n, and g_n are the complex Fourier components; and N is the uppermost harmonic in the applied voltage.

Examining these equations, it is clear that for $|n| \leq N$

$$t_n = p_n + g_n. \tag{5}$$

It is obvious that, if the sheaths are to be electrically symmetric, their individual waveforms will then be 1/2 of a fundamental RF period $(n = 1)$ out of phase. In addition, the polarity of one of the voltages needs to be reversed. The condition to have electrically symmetric sheaths is therefore

$$V_{\text{sp}}(\omega t) = -V_{\text{sg}}(\omega t + \pi). \tag{6}$$

Substituting (3) and (4) into (6), it is clear that the sheaths are electrically symmetric if and only if

$$p_n = -g_n e^{in\pi}. \tag{7}$$

$e^{in\pi} = 1$ whenever n is even. In similar, $e^{in\pi} = -1$ whenever n is odd. Using this result in (5) and (7) leads to the conclusion that, in order for the discharge to be electrically symmetric, $t_n = 0$ for even n and $t_n = 2p_n$ for odd n.

With this criterion, it is clear that the amplitude of the fundamental and the odd harmonics can be freely chosen. It is also clear that there can be no even harmonics contained in the waveform of the applied voltage. If an even harmonic is included, then the sheaths will not be electrically symmetric.

The consequences of this asymmetry effect are the topic of an upcoming publication, and the commercially useful aspects are patent pending (U.S. Provisional Application No. 61/038 263).

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